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Wheeler's delayed choice experiment, a well known manifestation of the complementarity principle, has proved somewhat difficult to physically interpret. We show that, restated in quantum field theoretic language, the experiment submits to a simple explanation: that wave- or particle-nature is imposed not at the slit plane but at the detector system. The intepretational difficulty conventionally encountered is due to the assumption of enforcement of complementarity at the former.

I. INTRODUCTION

As is well known, quantum mechanics manifests several non-classical phenomena arising because of superposition and entanglement [1]. One such is the delayed choice experiment (DCE) [2], which is essentially a dramatic realization of Bohr's complementarity principle (CP) [3], an interpretation of wave-particle duality of matter. As an illustration, let's consider a double-slit illuminated by a coherent source. An observer behind the slit plane is equipped with a dual detector system whereby he can observe the diffracted light with a screen or with two telescopes, one focussed on each slit. Detection at the screen produces a Young's double-slit pattern as each photon passes through both slits and interferes with itself. On the other hand, a detection at a telescope would imply the passage of the photon through that slit on which the telescope is focussed. This forces particle nature on the light and no Young's interference pattern is seen.

In DCE, the observer waits until after the light has passed the slit plane to decide whether he measures the wave- or particle-nature of the light. In the popular and scientific literature, it has provoked intriguing questions like: how does the light "decide" whether to pass through both slits or one of them in order to conform to the future decision of the observer? Does it do so via a backward-time effect? Or does it "know" what the observer will decide later on? In the following account, we present, using the formalism of quantum field theory, a conventional explanation of DCE in which this difficulty in physically interpreting the effect does not appear. Consideration of such issues in the foundations of quantum mechanics is relevant to the burgeoning field of quantum information [4], because they help to better visualize the nature and flow of information in quantum systems.

II. DERIVATION

We specialize to the single particle case the more general formalism used in the treatment of multiparticle interference [5]. Consider a source S that illuminates a diaphragm O perforated by two slits a and b. The state of the photon is given by the two-mode seperable state

$$|\Psi\rangle = |\text{vac}\rangle + \frac{\epsilon}{\sqrt{2}}(\hat{a}^\dagger e^{i\phi_A} + \hat{b}^\dagger e^{i\phi_B})|\text{vac}\rangle, \tag{1}$$

where ϵ determines the strength of the optical field, $\hat{X}^\dagger$ is the creation operator for the photon mode corresponding to slit X, ϕ_X is the phase factor associated with the mode X, $|\text{vac}\rangle$ is the underlying vacuum state. Hence, $|\Psi\rangle$ is a superposition of Fock states in modes a and b.

The light diffracts at the double-slit and falls on the screen to form an interference pattern, or perhaps on the aperture of a telescope to permit a path detection. The positive frequency part of the field at some point x on the detector is given by

$$E^{(+)}(x) = \hat{a}\exp(ik[d_s + d_{ax}]) + \hat{b}\exp(ik[d_s + d_{bx}]), \tag{2}$$

where $\hat{X}$ is the annihilation operator for the mode corresponding to slit X, k is the wave-number, d_s the distance from the coherent source to either slit and d_{Xx} the distance from slit X to point x on the screen. The probability $P(x)$ for detecting a photon at point x on the screen is given by $\langle E^{(-)}(x)E^{(+)}(x)\rangle$, where the angles $\langle\cdots\rangle$ represent expectation value in the state $|\Psi\rangle$. We find

$$P(x) \propto 1 + \cos(d_{ax} - d_{bx}), \tag{3}$$

if we set $\phi_A = \phi_B$, though this is not necessary to observe fringes. Eq. (3) is the usual far-field Young's double-slit pattern.

On the other hand, let us suppose the screen is replaced with a telescope focussed on one of the slits, say X. A detection with it is represented by the positive frequency part of the field operator, $E_X^{(+)}$, whose measurement implies the photon's passage through slit X. Then $E^{(+)}(x)$ is given by

$$E_a^{(+)}(x) \equiv \hat{a}\exp(ik[d_s + d_{a\xi}]) \qquad \text{or} \qquad E_b^{(+)}(x) \equiv \hat{b}\exp(ik[d_s + d_{b\eta}]). \tag{4}$$

Here $x = \xi$ is the position of the telescope focussed on slit a, and $x = \eta$, that of the telescope focussed on slit b. The probability for detection at either telescope is given by $\langle E_a^{(-)}(x)E_a^{(+)}(x)\rangle = \langle E_b^{(-)}(x)E_b^{(+)}(x)\rangle \propto \epsilon^2$. Therefore in this case, the probability for detection on either telescope is uniform and shows no fringes. This, as well as the result Eq. (3), is in keeping with what one expects for unentangled quantum systems on basis of the complementarity principle: that path information and first-order interference pattern are mutually exclusive.

We note that there is no explicit reference to time in the above calculation. Thus, there is no reason to expect that these results should not hold if the observer chooses to use the screen or the telescopes after the light crosses the slit plane. Therefore, the results derived above are sufficient to explain how delayed choice works.

III. PHYSICAL INTERPRETATION

Let us physically interpret the above results. The main point is that CP is not enforced on the photons at the slit plane. The photon passing through the slits does not need to "decide" whether to pass through both slits or one of them. It passes through both, irrespective of whether the observer subsequently measures position or momentum. The decision to manifest particle or wave nature occurs at the detector system according the observable, and hence allowed eigenstates, chosen by the observer. Amplitude information from both paths superpose at all x's. Even when the observer trains his telescope on one of the slits, amplitude information from both slits fall on the telescope aperture. But subsequently, the amplitude for one of the slits is filtered out by the telescope optics until only that for the focussed slit falls on the eye-piece. This, of course, is equivalent to measurement with one of the operators $E_X^{(+)}$.

Similar arguments apply also to a quantum eraser in which paths are potentially distinguishable via entanglement or one of the paths is unitarily marked (say, by a polarization rotator) [6]. This would force the expression of particle nature. However, as in DCE, the total information is not destroyed but remains hidden. Subsequent erasure of path information re-manifests correlated interference based on the superposition of amplitudes from both paths. However, we note that the status of complementarity is not always obvious in entangled systems. For example, a possible non-standard effect is discussed in Refs. [7,8].

The assumption implicit in DCE that in order to manifest particle nature the photon should have passed through only one of the slits turns out to be unnecessary for obtaining the required statistical predictions of the theory. It is this feature that frees the current explanation from having to invoke backward-time effects or cognitive interpretations. Suppose on the other hand that the slits are equipped with a suitable device to find out the slit the photon passes through. This, of course, forces a particle-like behaviour and destroys interference in the traditional sense of the complementarity principle. In this case there is a genuine lack of simultaneous information from both slits. Thus, we see that complementarity can be enforced both by an intermediary which-path measurement as well as at the final detection. These considerations suggest that wave-particle duality reflects a deeper information theoretic *and* quantum field theoretic nature of photons and, in general, matter.

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